PAPERS on Economics & Evolution

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0724

Patience, Fish Wars, rarity value & Allee effects.

by

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The *Papers on Economics and Evolution* are edited by the Evolutionary Economics Group, MPI Jena. For editorial correspondence, please contact: evopapers@econ.mpg.de

ISSN 1430-4716

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Patience, Fish Wars, rarity value & Allee effects

Reinoud Joosten*

October 18, 2007

Abstract

In a Small Fish War two agents interacting on a body of water have essentially two options: they can fish with restraint or without. Fishing with restraint is not harmful; fishing without yields a higher immediate catch, but may induce lower future catches.

Inspired by recent work in biology, we introduce into this setting rarity value and Allee effects. Rarity value means that extreme scarcity of the fish may affect its unit profit 'explosively'. An Allee effect implies that if the population size or density falls below a so-called Allee threshold, then only negative growth rates can occur from then on.

We examine equilibrium behavior of the agents under the limiting average reward criterion and the sustainability of the common-pool resource system. Assuming fixed prices at first, we show that patience on the part of the agents is beneficial to both sustainable high catches and fish stocks. An Allee effect can not influence the set of equilibrium rewards unless the Allee threshold is (unrealistically) high.

A price mechanism reflecting effects of the resource's scarcity, is then imposed. We obtain a rather subtle picture of what may occur. Patience may be detrimental to the sustainability of a high fish stock and it may be compatible with equilibrium behavior to exhaust the resource almost completely. However, this result does not hold in general but it depends on complex relations between the Allee threshold, the dynamics in the (interactive) resource and price systems, and the actual scarcity caused if the agents show no restraint.
Keywords: common pool resource systems, fish wars, limiting average rewards, sustainability, rarity value, Allee effect, stochastic games

1 Introduction

Levhari & Mirman [1980] introduced *The Great Fish War* to model strategic interaction between agents exploiting a natural renewable (or replenishable)

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resource. The game is a non-cooperative difference game, or a stochastic game (Shapley [1953]) with uncountable state and action spaces in the interpretation of Amir [2003], but it inspired a vast literature of especially *differential* fishery games.[1] The *Great Fish War* revealed that under various regimes of strategic interaction, the agents over-exploit the natural resource inducing effects similar to the 'tragedy of the commons' (Hardin [1968]).

The Small Fish War (Joosten [2007a]) is a game with frequency-dependent stage payoffs (Joosten [2003]), hence a stochastic game, and it has the following setup. Two agents possess the fishing rights to a body of water, and they have essentially two options, to fish with or without restraint. Restrictions in practice may take various forms, e.g., on catching seasons, on quantities caught, on technologies, e.g., boats, nets, allowed in catching. Essential for the intuition is that unrestrained fishing yields a higher immediate catch, but continued unrestrained fishing may lead to decreasing future catches. Restrained fishing by both agents is assumed to be sustainable.

In *the Small Fish War* agents maximize their average catches over an infinite time-horizon. In such a setting, a Folk Theorem result holds, i.e., any pair of individually rational rewards can be supported by an equilibrium such that a certain course of action is to be followed yielding exactly these rewards, called the equilibrium path. Any unilateral deviation from this equilibrium path is punished by the player being 'cheated', i.e., the latter adopts behavior in order to give the cheater a significant disadvantage relative to what the latter would have obtained on the equilibrium path.

The 'tragedy of the commons' does not seem inevitable, as Pareto-efficient outcomes can be sustained by subgame perfect equilibria. In a wide range of the parameter space of the model, the more the catches deteriorate due to over-fishing, the greater the gap between Pareto-efficient outcomes and the 'never restraint' outcome. We never found that 'perfect restraint' is Pareto-efficient *in the same parameter range*. Bulte *et al.* [1995] summarizing Holden [1994], partly attribute the ineffectiveness of fishery policies of the European Union on its biological rather than economic fundamental basis. A careful analysis of the *Small Fish War* shows that economic improvement may be feasible beyond the level reached by bluntly adhering to 'biologically optimal' restrictions. This then, might be the basis of a policy followed by the agents to select Pareto-efficient equilibria.

Courchamp *et al.* [2006] propose an interesting price-scarcity mechanism. Once a species becomes rare, its value may increase, this may induce greater incentives to exploit the natural resource, leading to even greater rarity, hence a higher value etc., etc. As a result, the population size or density may be pushed below a certain threshold beyond which only negative growth rates exist. Such an effect is called an Allee Effect in general, and

[1] For overviews and results on differential and stochastic games, see e.g., Dockner *et al.* [2001], Engwerda [2005], and e.g., Vrieze [1987], Thuijsman [1992], and Vieille [2000a,b].

because this one is caused by humans, the term Anthropogenic is added.

Moreover, Courchamp *et al.* [2006] list a number of real-world cases indicating that rarity value inducing the Anthropogenic Allee Effect (AAE) is not an armchair-scientist's oddity. Elsewhere, Berec *et al.* [2006] argue that the hazards of rarity value may be even greater than is presently recognized. Other Allee effects are known to exist, which in isolation do not harm a species at hand but in combination may affect the population dynamics in a disastrous way. Berec *et al.* [2006] state: '... *we suggest that multiple Allee effects could markedly affect the dynamics of the species concerned, and that the importance of multiple effects is concealed by the current lack of information about their prevalence*'.

Joosten [2007b] extends parts of the analysis of Courchamp *et al.* [2006] to multi-player interactive decision making by imposing 'rarity value' on a *Small Fish War*. It was shown that patient agents too, may over-exploit the resource. Lowest sustainable fish stock may imply highest sustainable stage payoffs which may imply Pareto-efficient outcomes which can be supported by subgame perfect equilibria.

Here, we examine several aspects not considered by Courchamp *et al.* [2006]. The latter contribution assumes implicitly that the agents only care for the present, or discount the future so heavily that (the infinite stream of) future catches evaluated at each point in time, are sufficiently similar to the prevailing, i.e., one-shot, situation. They also implicitly assume that the influence of the agents is significant in the sense that they can really harm the fish stock. Furthermore, to get into a scarcity region where prices indeed explode to levels dwarfing search costs (relatively) which induces very high unit profits, a region may have to be crossed in which profits are very low or even negative. Courchamp *et al.* [2006] offer little explanation how this can be accomplished, because for negative unit profits dominance relations in the one shot game are reversed compared to the positive unit profit case. The analysis in Courchamp *et al.* [2006] largely relies on considering unit profits. In this paper, we analyze *total profits*, i.e., unit profits times quantities, *averaged over an infinite period* by very *patient* agents in an *interactive decision making* framework with *bounded catching capacity*.

We find that a *Small Fish War* under 'rarity value' may very well exhibit the environmental effects sketched by Courchamp *et al.* [2006], i.e., 'no restraint' is the Pareto-efficient equilibrium, or if 'no restraint' would bring about the AAE, then behavior which induces fish stocks just above the Allee threshold may be consistent with equilibrium behavior. However, we also found parameter constellations leading to 'almost perfect restraint' as Pareto-efficient equilibria, to fish stocks which are near maximum level and to rewards which are slightly higher than 'perfect restraint' rewards.

Next, we review the small Fish War and then add an Allee effect. In Section 3 we examine the combined effects of rarity value and the Anthropogenic Allee Effect. Section 4 concludes with a discussion.

2 A quick review and an Allee effect

A *Small Fish War* is played by players A and B at discrete moments in time called stages. Each player has two actions and each stage each player independently and simultaneously chooses an action. We denote the action set of player A (B) by $J^A = \{0,1\}$ ($= J^B$) and $J \equiv J^A \times J^B$. Action 1 for either player denotes the action without or with very little restraint, e.g., catching with fine-mazed net or catching a high quantity. The other action denotes the action with some restriction, e.g., catching with wide-mazed nets or catching a low quantity. The payoffs at stage $t' \in \mathbb{N}$ of the play depend on the choices of the players at that stage, and on the relative frequencies with which all actions were actually chosen until then.

Let $h_{t'}^A = \left(j_1^A, ..., j_{t'-1}^A\right)$ be the sequence of actions chosen by player A until stage $t' \geq 2$, let $h_{t'}^B = \left(j_1^B, ..., j_{t'-1}^B\right)$ be defined similarly and let $q \geq 0$; define ρ_t recursively for $t \leq t'$ by

$$\rho_1 = \rho \in [0,1]\,, \text{ and } \rho_t = \frac{q+t-1}{q+t}\rho_{t-1} + \frac{1}{q+t}\left(\frac{j_{t-1}^A + j_{t-1}^B}{2}\right). \tag{1}$$

Taking $q \gg 0$ serves to moderate 'early' effects. Note that for the long run the choice of numbers ρ and q is irrelevant.

At stage $t \in \mathbb{N}$, the players have chosen action sequences h_t^A, h_t^B which induce the number ρ_t. The latter number determines the state in which the play is at stage t. Slightly more formal, we say that the play at stage $t \in \mathbb{N}$ is in state $s_t \equiv \rho_t$. Observe that Eq. (1) implies that there are three possible successor states s_{t+1} to state s_t depending on the action pair chosen at t.

At each stage a bi-matrix game is played, and the choices of the players at that stage determine their stage payoffs. Let

$$A = B^\top = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Then, for given $\mu_t \in [0,1]$ at stage $t \in \mathbb{N}$, the stage payoffs are given by

$$\mu_t\left(A,B\right) = \begin{bmatrix} a\mu_t, a\mu_t & b\mu_t, c\mu_t \\ c\mu_t, b\mu_t & d\mu_t, d\mu_t \end{bmatrix}.$$

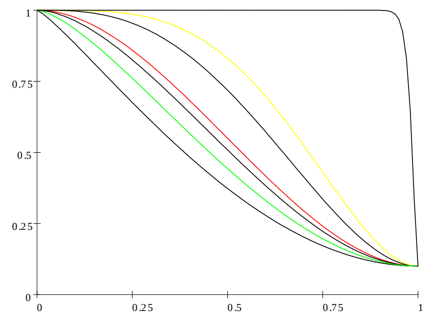
Here, μ_t may be interpreted as a measure for the present fish stock; if player A chooses action 0 and B chooses action 1, A's stage payoff is $b\mu_t$ and B's is $c\mu_t$. We assume that fishing without restraint yields a higher catch in any current stage than fishing with restraint, hence $a < c$, $b < d$. We assume that two-sided catching without restraint yields higher immediate payoffs than two-sided catching with restraint, i.e., $a < d$. The unique stage-game equilibrium is the strategy pair in which both players use action 1.

Now, we will specify μ_t. At stage $t \in \mathbb{N}$, the play is in state $s_t = \rho_t$, then the **normalized fish stock** is given by

$$\mu_t \equiv 1 + (1 - \underline{m}) \left[\frac{n_2}{n_1 - n_2} \rho_t^{n_1} - \frac{n_1}{n_1 - n_2} \rho_t^{n_2} \right], \tag{2}$$

where $\underline{m} \in [0, 1]$ represents the minimal stock due to overexploitation by the agents, and $n_1 > n_2 > 1$. So, (2) determines how the fish stock deteriorates from its maximum due to fishing without restraint.[2] For increasing n_1, the deterioration of the fish stock near its maximum, is less and less noticeable; as a consequence the descent 'later on' must be steeper, because the minimal stock is $\underline{m}$. For $\underline{m} = 1$, we have a standard repeated game.

Below, (2) is visualized for $\underline{m} = 0.1$, $n_2 = n_1 - 1$, and different values of n_1; the greater n_1, the higher the corresponding curve. For the six lower curves n_1 lies between 2.2 and 5; the highest curve has $n_1 = 100$. For the latter value of n_1, noticeable effects on the fish stock are to be found when e.g., *both* agents fish without restraint for approximately 90% of the time.



If both agents never show restraint, then the associated long run stage payoffs are $d\underline{m}$; if they show perfect restraint, then the associated long run stage payoffs equal a. In the remainder we make the following assumptions.

Remark 1 *'Never restraint' gives at most half the long-run stage payoffs associated with 'perfect restraint', i.e.,* $d\underline{m} \leq \frac{a}{2}$*; the sharpest decline of the stock occurs at* $\rho^* = \left(\frac{n_2-1}{n_1-1}\right)^{\frac{1}{n_1-n_2}} \in \left[\frac{1}{4}, \frac{3}{4}\right]$.

The latter part seems reasonable as for large n_1, n_2 unrestrained fishing can go on for quite a while without having a noticeable effect on the environment. Conversely, for $n_1 \approx n_2$ or fixed n_1 and $n_2 \downarrow 1$, the environment is extremely sensitive to the slightest instance of fishing without restraint.

[2]'Real-world' alternatives to (2) can be fit in easily provided they are **continuous**.

2.1 Strategies and rewards

At stage t, both players know the current state and the history of play, i.e., the state visited and actions chosen at stage $u < t$ denoted by $\left(s_u, j_u^A, j_u^B\right)$. A **strategy** prescribes at all stages, for any state and history, a mixed action to be used by a player. The sets of all strategies for A respectively B will be denoted by $\mathcal{X}^A$ respectively $\mathcal{X}^B$, and $\mathcal{X} \equiv \mathcal{X}^A \times \mathcal{X}^B$. The payoff to player k, $k = A, B$, at stage t, is stochastic and depends on the strategy-pair $(\pi, \sigma) \in \mathcal{X}$; the **expected stage payoff** is denoted by $R_t^k\,(\pi, \sigma)$.

The players receive an infinite stream of stage payoffs during the play, and they are assumed to wish to maximize their average rewards. For a given pair of strategies (π, σ), player k's **average reward,** $k = A, B$, is given by $\gamma^k\,(\pi, \sigma) = \liminf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} R_t^k\,(\pi, \sigma)$; $\gamma\,(\pi, \sigma) \equiv \left(\gamma^A\,(\pi, \sigma), \gamma^B\,(\pi, \sigma)\right)$. It may be quite hard to determine the **set of feasible (average) rewards** F, directly. Here, we focus on rewards from strategies which are pure and jointly convergent. Then, we extend our analysis to obtain larger sets of feasible rewards.

A strategy is **pure**, if at *each* stage a **pure action** is chosen, i.e., the action is chosen with probability 1. The set of pure strategies for player k is $\mathcal{P}^k$, and $\mathcal{P} \equiv \mathcal{P}^A \times \mathcal{P}^B$. The strategy pair $(\pi, \sigma) \in \mathcal{X}$ is **jointly convergent** if and only if $z \in \Delta^{m\times n}$ exists such that for all $\varepsilon > 0$, $(i, j) \in J$:

$$\limsup_{t\to\infty} \Pr_{\pi,\sigma}\left[\left|\frac{\#\{j_u^A = i \text{ and } j_u^B = j |\ 1 \leq u \leq t\}}{t} - z_{i+1,j+1}\right| \geq \varepsilon\right] = 0,$$

where $\Delta^{m\times n}$ denotes the set of all nonnegative $m \times n$-matrices such that the entries add up to 1; $\Pr_{\pi,\sigma}$ denotes the probability under strategy-pair (π, σ). $\mathcal{JC}$ denotes the set of jointly-convergent strategy pairs. Under such a pair of strategies, the relative frequency of action pair $(i, j) \in J$ converges with probability 1 to $z_{i+1,j+1}$ in the terminology of Billingsley [1986, p.274].

The **set of jointly-convergent pure-strategy rewards** is given by

$$P^{\mathcal{JC}} \equiv cl\left\{\left(x^1, x^2\right) \in \mathbb{R}^2 |\ \exists_{(\pi,\sigma)\in\mathcal{P}\cap\mathcal{JC}} : \left(\gamma^k\,(\pi, \sigma), \gamma^k\,(\pi, \sigma)\right) = \left(x^1, x^2\right)\right\},$$

where $cl\ S$ is the closure of the set S. The interpretation of this definition is that for any pair of rewards in this set, we can find a pair of jointly-convergent pure strategies that yield rewards arbitrarily close to the original pair of rewards. With respect to jointly-convergent strategies, Eq. (2) and the arguments presented imply that

$$\lim_{t\to\infty} \mu_t\,(a_{ij}, b_{ij}) = \left(1 + (1 - \underline{m})\left(\frac{n_2}{n_1 - n_2} Z^{n_1} - \frac{n_1}{n_1 - n_2} Z^{n_2}\right)\right)(a_{ij}, b_{ij}),$$

where $Z \equiv z_{22} + \frac{1}{2}\,(z_{12} + z_{21})$. So, the bi-matrices $\mu_t\,(A, B)$ 'converge' in the long run, too.

Let $\varphi(z) \equiv \sum_{(i,j)\in J} z_{i+1,j+1}\left(\lim_{t\to\infty} \mu_t\,(a_{ij}, b_{ij})\right)$. The interpretation of $\varphi(z)$ is that under jointly-convergent strategy pair (π, σ) the relative frequency of action pair $(i, j) \in J$ being chosen is $z_{i+1,j+1}$ and each time this occurs the players receive $\lim_{t\to\infty} \mu_t(a_{ij}, b_{ij})$ in the long run. Hence, the players receive an average amount of $\varphi(z)$. So, $\gamma(\pi, \sigma) = \varphi(z)$. Several algorithms exist to compute $P^{\mathcal{JC}}$ for linear two-person FD-games. For Small Fish Wars, we designed a new, more general one. We refer to Figure 2 for an illustration.[3]

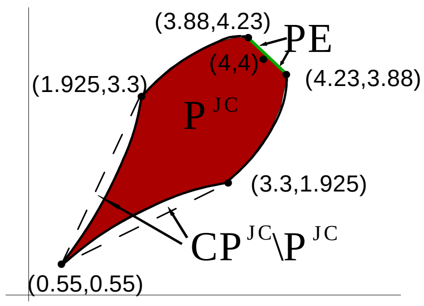


Figure 1: Here, $\underline{m} = 0.1$, $n_1 = 3$, $n_2 = 2$, $a = 4$, $b = 3.5$, $c = 6$, $d = 5.5$. The red area denotes $P^{\mathcal{JC}}$; $CP^{\mathcal{JC}}$ is the convex hull of $P^{\mathcal{JC}}$. PE denotes Pareto efficient rewards in $P^{\mathcal{JC}}$; clearly, $(4, 4)$ is not Pareto efficient.

2.2 Threats and equilibria

The strategy pair (π^*, σ^*) is an **equilibrium**, if no player can improve by unilateral deviation, i.e., $\gamma^A(\pi^*, \sigma^*) \geq \gamma^A(\pi, \sigma^*)$, $\gamma^B(\pi^*, \sigma^*) \geq \gamma^B(\pi^*, \sigma)$ for all $\pi \in \mathcal{X}^A, \sigma \in \mathcal{X}^B$. An equilibrium is called **subgame perfect** if for each possible state and possible history (even unreached states and histories) the subsequent play corresponds to an equilibrium, i.e., no player can improve by deviating unilaterally from then on. In the construction of equilibria for repeated games, 'threats' play an important role. A threat specifies the conditions under which one player will punish the other, as well as the subsequent measures.

We call $v = (v^A, v^B)$ the **threat point**, where $v^A = \min_{\sigma\in\mathcal{X}^B} \max_{\pi\in\mathcal{X}^A} \gamma^A(\pi, \sigma)$, and $v^B = \min_{\pi\in\mathcal{X}^A} \max_{\sigma\in\mathcal{X}^B} \gamma^B(\pi, \sigma)$. So, v^A is the highest amount A can get if B tries to minimize his average payoffs. Under a pair of **individually rational** (feasible) rewards each player receives at least the

[3] We used Excel to compute and visualize more than 12000 jointly-convergent pure-strategy rewards. The Excel figure contains holes, the real set is dense in $\mathbb{R}^2$ as presented.

threat-point reward. We can now present the major result of Joosten [2007] generalizing the one in Joosten *et al.* [2003].

Theorem 1 *Each pair of rewards in the convex hull of all jointly-convergent pure-strategy rewards giving each player strictly more than the threat-point reward, can be supported by a subgame-perfect equilibrium.*

The following consequence of this result is illustrated in Figure 3.

Corollary 2 *Let* $E' = \{(x,y) \in P^{\mathcal{JC}} | \ (x,y) > v\}$, *then each pair of rewards in the convex hull of cl* E' *can be supported by an equilibrium. Moreover, all rewards in* E' *can be supported by a subgame-perfect equilibrium.*

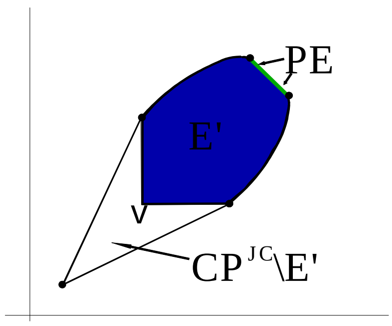


Figure 2: The blue area represents equilibrium rewards. The set of Pareto-efficient equilibria in E' is denoted by a green line segment, note that $v \approx (1.925, 1.925)$.

In the example used for Figures 1 and 2, Pareto-efficient equilibria

- yield combined rewards which are slightly (1.4%) higher than the combined 'perfect restraint' equilibrium rewards $(4,4)$, yet *more than seven times* the combined 'never restraint' rewards as $\frac{a}{dm} = \frac{80}{11}$;
- induce play in which both players simultaneously show restraint for about 85.6% of the stages; otherwise precisely one shows restraint.

Under Remark 1, the difference between the 'never restraint' rewards and the symmetric Pareto-efficient equilibrium rewards increases as $\underline{m}$ decreases, whereas the difference between the symmetric Pareto-efficient equilibrium rewards and the 'perfect restraint' rewards decreases.

2.3 Addition of an Allee effect

We now introduce the notion of an Allee effect into the small Fish War with constant prices. The following quote may be found in Berec *et al.* [2006]: *'Allee effects occur whenever fitness of an individual in a small or sparse population decreases as the population size or density declines'.* Courchamp *et al.* [2006] explain: *'Populations suffering from Allee effects may exhibit negative growth rates at low densities, which drives them to even lower densities and ultimately to extinction'.* Berec *et al.* [2006] also define an Allee threshold as the '*critical population size or density below which the per capita population growth rate becomes negative'.*

Let therefore Th denote an Allee threshold measured in the same dimension as the fish stock. We formalize the explanations above by

$$\mu_t = 1 + (1-\underline{m})\left[\frac{n_2}{n_1-n_2}\rho_t^{n_1} - \frac{n_1}{n_1-n_2}\rho_t^{n_2}\right] \text{ if } \mu_s \geq Th \text{ for all } s \leq t,$$
$$\frac{\mu_s - \mu_{s-1}}{\mu_{s-1}} < \theta < 0 \text{ and all } s \geq s' \text{ such that } \mu_{s'} < Th. \tag{3}$$

The second part of (3) does not specify what happens, but it captures the Allee effect in a rather general manner. For our purposes here, this will do, we bounded $\frac{\mu_s - \mu_{s-1}}{\mu_{s-1}}$ away from zero because then the population becomes extinct *in finite time*. Hence, if under strategy pair (π, σ) the fish stock at any point in time drops below the Allee threshold, then $\lim_{t\to\infty} \mu_t = 0$; we normalize the associated rewards to $\gamma(\pi, \sigma) = (0, 0)$ and call them Collapse Rewards, since the economic as well as the resource system break down.

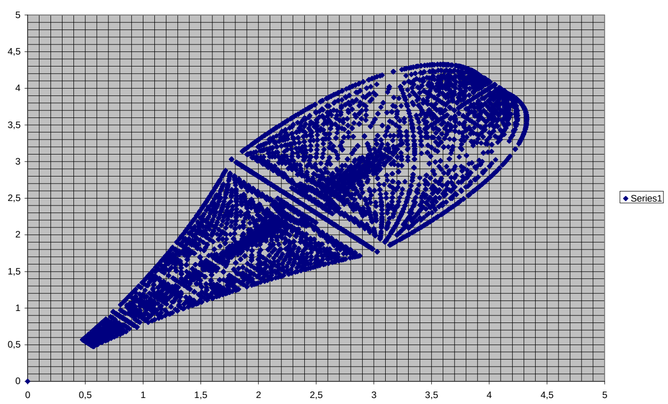


Figure 3: $P^{\mathcal{JC}}$ for $\underline{m} = 0.01$ and $Th = 0.1$. Jointly-convergent pure strategies rewards inducing fish stocks below Th are normalized to $(0, 0)$.

Figure 3 visualizes the set of jointly-convergent pure-strategy rewards which can be achieved under the Allee effect. The threat point is unaffected by the Allee effect, it is by the lower minimal fish stock, though. The new threat point is given by $v \approx (1.7675, 1.7675)$. So, the equilibrium rewards are quite far removed from the Collapse Rewards, and Pareto-efficient ones even furthest. This means self-interested rational agents will behave in the interest of the environmental system in order to guarantee high fish stocks. They stay far above the Allee threshold in doing so.

It should be noted that although the set of equilibrium *rewards* is unaffected by the introduction of the Allee effect into the model, the set of equilibrium *strategies* is reduced. All equilibria having an equilibrium path inducing fish stocks below the threshold at some point in time in the original model, are not equilibria in the modified setting as they will yield the Collapse Rewards. The only way in which the Allee threshold has any influence on the set of equilibrium rewards for the model with the parameters as presented, is to increase the threshold to at least 0.36. We have not seen any actual numbers for Allee thresholds in the works studied for this paper, but in our non-expert opinion, the latter number seems unreasonably high.

3 Rarity value and averaging

The *Small Fish War* and its extension presented here implicitly model a situation in which agents sell their catches at a competitive market while incurring fixed unit search costs, at least fixed with respect to the scarcity of the resource in their fishing environment. Alternatively, if neither prices on the market, nor search costs are fixed, then one can regard the model as pertaining to a situation in which unit prices go up approximately in the fashion as the unit search costs do.

In some cases, fishermen extract less and less in quantities, nevertheless obtain higher and higher revenues. An 'ongoing' real-world example of such an anomaly might be the market for blue fin tuna, fish stocks seem to have decreased by 80 percent in the last five years, but prices have sky-rocketed especially in the Far East (e.g., Veldkamp [2007]). In economics a range of anomalies are known as Veblen and status goods (cf., e.g., Leibenstein [1950]), but these are hardly ever linked to exhaustible resources, let alone animal species facing extinction.

Courchamp *et al.* [2006] model and analyze the latter aspect, and cite real-world cases in which prices for certain rare animals increase more than search costs leading to the extinction of these endangered species. Below, we have adapted a poignant figure from Courchamp *et al.* [2006]. The red curve represents the search costs of catching a certain unit of fish as a function of its availability, to be captured by the fish stock μ in our model (on the horizontal axis). The explicit formula for the unit costs is given by

$$c(\mu) = \frac{4}{3.75}\left(12 - 12\mu + \frac{1}{\mu^{1.5}}\right),$$

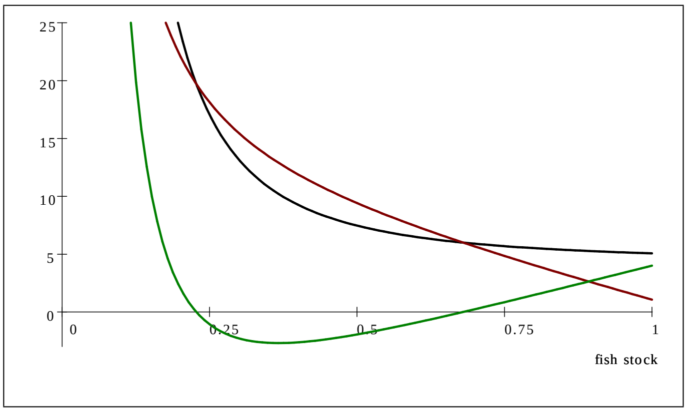
where $c(\mu)$ is the unit costs given the (normalized) fish stock μ. The unit costs increase as the species becomes rarer, i.e., μ becomes smaller. The unit prices, represented by the black curve, remain nearly constant between $\mu = 1$ and $\mu = 0.6$, but for lower availability of the fish stock they go up sharply. The specifics of the formula for the unit prices are

$$p(\mu) = \frac{4}{3.75}\left(4 + 0.75\frac{1}{\mu^2}\right),$$

where $p(\mu)$ is the unit price to be obtained on the market as a function of the (normalized) fish stock μ. As unit profits equal unit price minus unit costs, the resulting unit profit curve, drawn in blue-green, is given by[4]

$$\pi(\mu) = \frac{4}{3.75}\left(-8 + 12\mu + 0.75\frac{1}{\mu^2} - \frac{1}{\mu^{1.5}}\right). \tag{4}$$

It is readily confirmed that $\pi(1) = 4$ and that $\lim_{\mu\downarrow 0}\pi(\mu)$ does not exist.



So, unit profits decrease as fish stocks decrease from maximal level, because the unit price remains almost constant, but unit search costs increase steadily. If the fish stock continues to fall below approximately 0.675, unit profits become negative, i.e., the agents would incur losses by catching fish. However, if the fish stock would fall below approximately 0.228, then the unit price driven by scarcity would exceed unit costs again. Moreover, increasing scarcity causes the unit price to increase more than the unit costs from then on. According to Courchamp *et al.* [2006] such a profit-scarcity relationship spells doom for the survival of the animal species.

Below, we present a figure (generated by Excel) in which the relation between unit profits and the availability of fish used to generate the adaptation from Courchamp *et al.* [2006] above, is added to the functions governing the computations for original Fish War.[5] Since, a range in which unit profits

[4] To allow easy comparison with the previous results, numbers have been scaled, i.e., unit profit is 4 if $\mu = 1$. The matrix entries have been divided by 4 to compensate for this.

[5] The set looks like a fish, skeptics may obtain the Excel file from the author on request.

are negative exists, we find negative average rewards whereas in the original game all rewards were positive. For fish stocks with $\mu < 0.228$ profits increase steadily as fish stock declines.

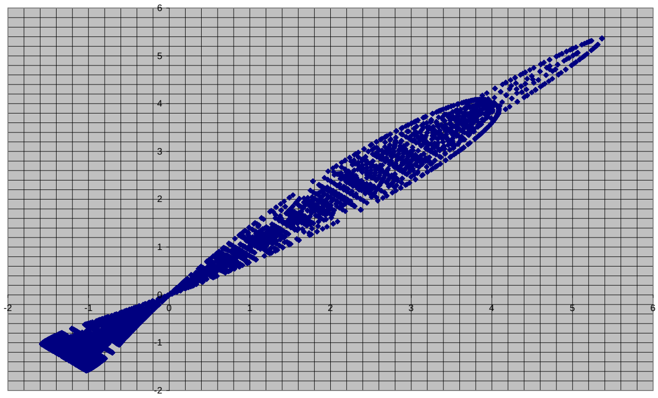


Figure 4: $P^{\mathcal{JC}}$ for the same parameters as before, but with profit-scarcity relationship given by π. Perfect restraint yields $(4,4)$, but rewards (the true set is dense) in the 'beak' exist Pareto dominating $(4,4)$ significantly.

We have the following result pertaining to the threat point $v = \left(v^A, v^B\right)$.

Lemma 3 *For the parameters and the profit-scarcity relationship (4) we have* $v^A, v^B \leq \overline{v} = -0.70445$.

Given $v \leq \overline{v}$, the following is trivially valid in view of Theorem 1.

Corollary 4 *Let* $E' = \{(x,y) \in P^{\mathcal{JC}} | \ (x,y) > \overline{v}\}$, *then each pair of rewards in the convex hull of cl* E' *can be supported by an equilibrium. Moreover, all rewards in* E' *can be supported by a subgame-perfect equilibrium.*

The Pareto optimal equilibrium seems to be 'no restraint' which yields approximately $(5.36, 5.36)$ and induces fish stock equal to $\underline{m} = 0.1$. Hence, there is a clear price effect which spells doom for the sustainability of high catches, as it overcompensates the effect of low catches.

3.1 Rewards, rarity value and alternative parameter choices

Having just found an example confirming the scenario sketched by Courchamp *et al.* [2006], the natural question arises about the generality of such events. In the remainder of this subsection we identify two factors which appear to be decisive for the sustainability of the resource system under the novel ideas regarding 'rarity value' and the AAE inspired by Courchamp *et* *al.* [2006]. The first factor is related to the 'actual harm' persistent unrestricted catching causes, i.e., the minimal stock level $\underline{m}$. The second one is that that unit profits are not the real issue, but (long-term average) total profits are under the evaluation criterion chosen.

First, if we change the minimal stock level to $\underline{m} = 0.12$, the 'beak' disappears. As in the original Small Fish War, the Pareto optimal equilibria give rewards quite close to 4 and a large proportion of the catches must be restrained. Several equilibrium rewards may be obtained in two different ways. One way is to obtain the equilibria by both agents being fairly modest in the propensity to catch without restraint, the other is to obtain the same rewards by both agents catching without restraint quite ruthlessly.

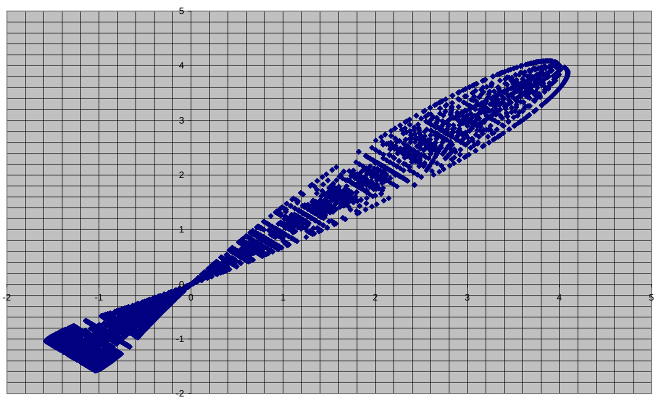
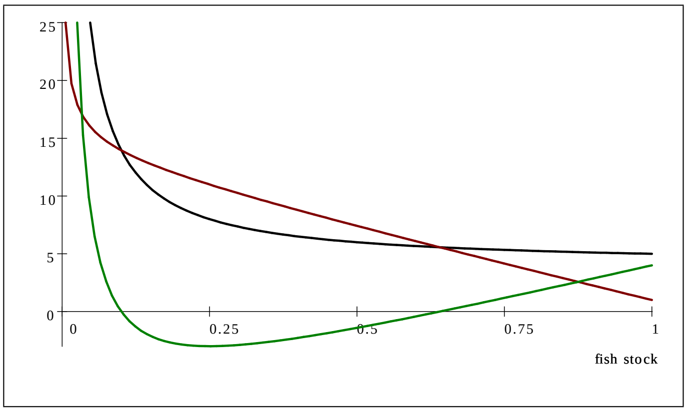


Figure 5: $P^{\mathcal{JC}}$ for $\underline{m} = 0.12$ instead of 0.1. The 'beak' disappears and the Pareto optimal equilibria are near the perfect restraint equilibrium again.

Now, we change the equations giving the profit-scarcity function. Let, alternatively, the unit profit curve be given by

$$\pi'(\mu) = p(\mu) - c(\mu) = \frac{4}{3.75}\left[\left(4 + 0.75\frac{1}{\mu}\right) - \left(12 - 12\mu + \frac{1}{\mu^{0.5}}\right)\right].$$

The connection to the previous formula is that the qualitative features of the corresponding curves are similar. There is a slight difference in levels and the intersection points of the blue-green curve are somewhat different. The most significant difference for the analysis is that for $\underline{m} = 0.1$, the average rewards to both agents $-0.262\,65$.



So, the limited catching capacities of the agents actually wards off the threat of extinction. Even if they were unlimited, the price effects do not dominate the quantity effects sufficiently to obtain sufficiently high long-run average profits. The setting with regard to the limiting average rewards changes as the figure below indicates. Figures 4, 5 and 6 are similar except for the beak shaped area representing rewards which Pareto-improve significantly with respect to the 'perfect restraint' outcome.

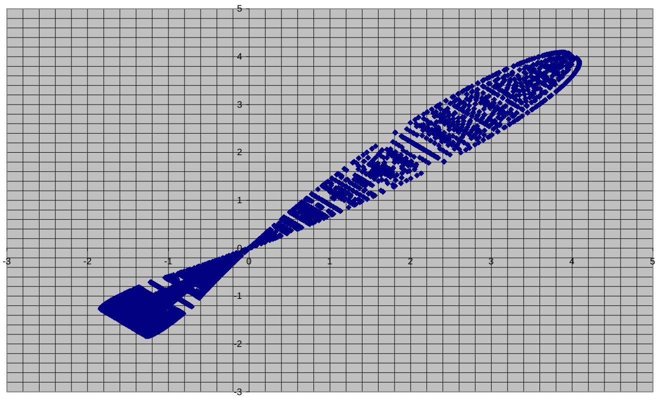


Figure 6: For $\underline{m} = 0.1$ and π', the beak-shaped area disappears.

We have generated similar visualizations of the sets of rewards for *smaller values* of $\underline{m}$. We found that for steadily decreasing values of $\underline{m}$ 'no restraint' yields steadily higher rewards. For the sake of comparison we toyed with other parameters of the model too, but the 'rarity value' effect prevails in similar fashions throughout the variants examined. A visualization is given in Figure 7.

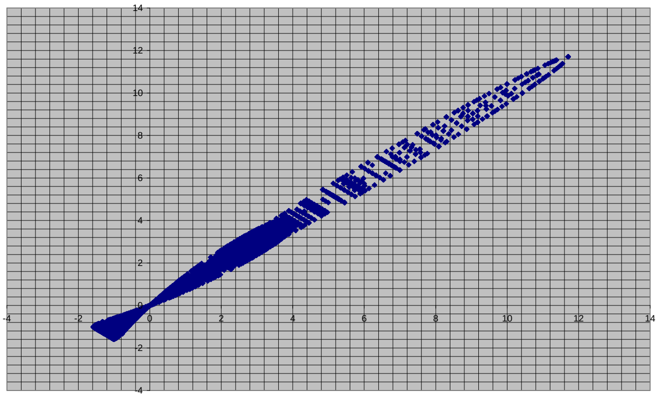


Figure 7: $P^{\mathcal{JC}}$ for $\underline{m} = 0.06$ and all other parameters as before. The Pareto efficient equilibrium is $(11.705, 11.705)$.

3.2 The Anthropogenic Allee Effect

As we have seen in the *Small Fish War* with constant prices to which an Allee effect was added, a subset of the jointly-convergent pure-strategy rewards is cut off. Since Allee effects only occur if the fish stock drops below a certain threshold only lower left-hand-side rewards in $P^{\mathcal{JC}}$ are affected there. However, due to effects of 'rarity value' the area which is cut off under an addition of an Allee effect may be expected to lie in the upper right-hand corner. Hence, if this is confirmed, the set of equilibria is likely to be involved as well.

Comparing the Figures 4 and 8, we find that part of the 'beak' has disappeared in the latter as anticipated. The Anthropogenic Allee Effect reduces the set of rewards situated in the 'beak'. Since the upper bound for the threat point is not affected by the AAE, all rewards to the 'north-west' of $\overline{v}$ in Figure 8 are equilibrium rewards. Hence, the AAE eliminates the unique Pareto-efficient equilibrium reward in $P^{\mathcal{JC}}$ along with a set of rewards forming a considerable Pareto-improvements compared to 'perfect restraint' of the same model without the AAE.

4 Conclusions

Courchamp *et al.* [2006] introduce 'rarity value' in a common-pool resource system in which unit profits of agents in an economic system dependent on this resource, increase to very high levels as the availability of the species goes to zero. There is a potential dangerous side to such a system as the propensity to exploit the resource increases as its scarcity increases which increases the propensity to exploit and so on. An important role is attributed

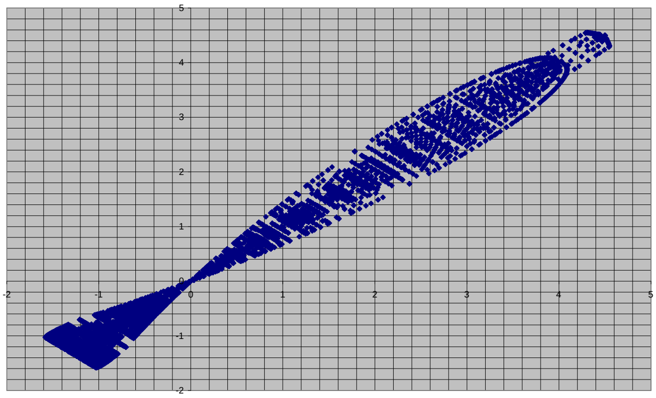


Figure 8: Here, we have $m = 0.1$ and $Th_{AAE} = 0.11$, other parameters where taken as in the earlier figures. The set of jointly-convergent pure-stratgy rewards is cut off in the 'beak'.

to Allee effects, i.e., once the population size or density of the resource falls below a so called Allee threshold, only negative growth rates are possible.

For the sake of comparison, we generated and analyzed two variants of a *Small Fish War* (Joosten [2007]). One variant takes unit profits as fixed and an Allee effect is then added. Here, high sustainable yields can only be accomplished if the agents preserve the resource at stock levels well above the minimum. The Allee effect does not influence the set of equilibrium rewards unless the Allee threshold is unrealistically high. Hence, self-interested rationality and the sustainability of the resource may go together very well.

The other variant incorporates 'rarity value' and then the Anthropogenic Allee Effect (AAE) is added. 'Rarity value' implies that the highest *sustainable* unit profits in the low-stock-level range are attained at the maximum of the AAE threshold and the 'no restraint' stock level. The analysis considers the combined effects of (decreasing) quantity and (increasing) unit profits evaluated in the long run.

It is by no means a general result that the rewards associated with this maximum constitute even a Pareto-improvement over the 'perfect restraint' equilibrium. If it is not, we have a similar result as in the fixed unit profits variant: self-interest and sustainability provide no tensions. However, if these rewards associated with this maximum are sufficiently high, they constitute a unique Pareto-efficient equilibrium reward. Hence, the economic system and the resource system have conflicting interests. Highest sustainable equilibrium rewards can only be accomplished by reaching the lowest possible sustainable fish stock on purpose. There is also a range of parameters which generate results which are in-between the two extremes just mentioned. Only a careful analysis can reveal to which extreme case the model and its result belong. We have even found that in some parameter constellations, one and the same Pareto-efficient equilibrium reward may be obtained in radically different manners. In one the reward is attained by showing no restraint at all, in the other they show a rather high degree of restraint.

In these respects, we add some novel aspects and new insights to the frameworks of Joosten [2007a,b] as well as Courchamp *et al.* [2006].

5 Appendix

Proof of Lemma 3 There is not much to go by to compute the threat point in FD-games. We have used a variety of techniques elsewhere (e.g., Joosten *et al.* [2003], Joosten [2007a,b,c]). So, we present a more modest claim formulated in the statement of the lemma. We will show that player A cannot obtain more than $\overline{v}$ against a fixed strategy $\overline{\sigma}$ given by

$$\overline{\sigma}_t = \begin{cases} 1 & \text{if } t = 1 \\ j \in J \backslash \{j_{t-1}^A\} & \text{otherwise.} \end{cases}$$

Observe that in the long run, $\rho_t \to \frac{1}{2}$ and $\mu_t \to \frac{1}{2}\left(1 + \underline{m}\right)$. Then, the long-run unit-profit under any strategy used by player A equals $\pi_t\left(\frac{1}{2}\left(1 + \underline{m}\right)\right) < 0$. Hence, the stage payoffs, i.e., the total profits at that stage, are equal to at most $\min\left\{\frac{a}{4}, \frac{b}{4}\right\}\left(\frac{1}{2}\left(1 + \underline{m}\right)\right)\pi_t\left(\frac{1}{2}\left(1 + \underline{m}\right)\right) = \overline{v}$. This in turn implies that the average rewards can not exceed $\overline{v}$, either. So, the column player clearly possesses a strategy to keep the row player's rewards at at most $\overline{v}$. This obviously implies that $v^A \leq \overline{v}$. ■

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